1993-94 School Year

Mathematics 30 Information Bulletin

Diploma Examinations Program

Students First!
Student Evaluation

Alberta
EDUCATION

This document was written primarily for:

Students	
Teachers	✓
Administrators	
Parents	
General Public	
Others (Specify)	

Contents

Mathematics 30 Information Bulletin, Diploma Examinations Program, 1993–94 School Year

1994 Diploma Examinations Schedule

Date	Time
Friday, January 28	9 to 11:30 a.m.
Monday, June 27	1 to 3:30 p.m.
Wednesday, August 17	1 to 3:30 p.m.

Note: The diploma examinations are designed as 2.5 hour examinations. It is expected that this is adequate time for students to finish. However, for those students who need it, an extra 0.5 hour is available.

1994 Marking Information

The written-response portion of the Mathematics 30 diploma examinations is marked by classroom teachers.

To qualify as markers, teachers must
- be recommended by their superintendents,
- have taught the subject for two or more years,
- be currently teaching the subject, and
- have an Alberta Permanent Professional Certificate.

Student Evaluation particularly needs teachers who can mark examinations written in French.

Teachers who wish to be recommended as markers for the January 1994 examination should contact their superintendents before September 30, 1993.

Teachers who wish to be recommended as markers for the June and August 1994 examinations should contact their superintendents before March 2, 1994.

The 1994 examinations will be marked on the following dates:

January 1994 Administration	Jan. 31 to Feb. 5
June 1994 Administration	July 4 to 9
August 1994 Administration	August 19 to 20

1993–94 Field Testing and Item Writing

As the need arises for teachers to participate in field testing and item writing, letters are sent to superintendents requesting their nominations. Teachers who are interested in these activities should let their superintendents know early in the school term.

Directions for the 1994 Field Tests

The 1994 Mathematics 30 field tests will continue to include questions that require students to describe their method of problem solving and to communicate their descriptions of mathematical definitions and situations. Furthermore, field tests will also include items that assess how well students have achieved the general learner expections stated in the *Mathematics 30 Course of Studies*. Students will be challenged to show they understand mathematical concepts and can apply them in real-life situations.

Use of Scientific Calculators on Examinations

The term *scientific calculator* includes all hand-held devices designed for mathematical computations. These scientific calculators may have graphing capabilities, built-in formulas, mathematical functions, or other programmable features. Computers or devices with a primary function of random access storage do not fit the

definition of scientific calculator. (Please refer to Appendix A for the policy statement on the use of scientific calculators on diploma examinations.) The *Mathematics 30 Course of Studies* indicates that

> "Skills specifically related to the use of *technology* identify areas in which scientific calculators and/or computer technology are applied by students as tools to be used for calculations, manipulation or graphing, or to aid in the analysis of problems. Technological expectations are defined explicitly throughout the learner expectations. In many cases, a particular technology is indicated for investigation or analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving."
>
> (From *Mathematics 30 Course of Studies*, p. 5).

The diploma examinations are constructed to ensure that the use of particular scientific calculators does not advantage or disadvantage individual students. Students do not need a graphing calculator to write the examination. However, we do expect that students have used graphing calculators and/or computer graphing utilities in developing their understanding of mathematical concepts. Students who have used this technology in their mathematics class have an advantage over those students who have not used it before.

Students should be made aware of this as early as possible in the school term to ensure they are able to use the scientific calculator of their choice when writing the diploma examination.

Students should also be made aware of the Examination Rules, Grade 12 Diploma Examinations (see Appendix B of this bulletin), one of which states that students must not bring notes stored in electronic devices into the examination room.

Not Permitted

In Mathematics 30, notes include any form of written text or algebraic text. One example of notes that are not permitted in calculators is:

If $e = 1$, then the quadratic relation is a parabola.

A second example of a note is:

The cosine of $\frac{\pi}{3}$ is $\frac{\sqrt{3}}{2}$.

or:

$\cos \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Permitted

The Examination Rules also state that "Calculator programs designed to perform mathematical computations or those designed to assist students in graphing are not classified as notes." An example of a program that would assist students with graphing is the graphing program for graphing Quadratic Relations found in the *Mathematics 30/33 Interim Teacher Resource Manual*. A second example of a program that students could use is a program designed to solve the quadratic formula.

Examiners' Reports

We send examiners' reports to schools following the administration of the January and June examinations. These reports, to be used primarily by teachers, briefly outline the statistical data from the examination administration and provide a diagnostic overview of student performance on each examination. Student Evaluation welcomes comments from teachers about how to increase the usefulness of these reports. Please direct your comments to the Mathematics 30 examination manager.

School and Jurisdiction Statistical Reports

We send superintendents and principals detailed statistical reports on how well the students in their school district did on the Mathematics 30 examination. We expect that teachers will use these data to reflect on the areas of the program that their students did well in and those areas where student performance was poor.

Annual Report

An annual report that summarizes results from the January, June, and August diploma examination administrations is published each year. The purpose of this report is to inform educators and the public about student achievement in relation to provincial standards.

Standards

Provincial standards help to communicate how well students need to perform to be judged as having achieved the learnings specified for Mathematics 30. According to the *Mathematics 30 Course of Studies*, student learnings refer to specific knowledge, skill, and attitude expectations. These learnings are amplified in Appendix C, Mathematics 30 Curriculum Standards, of this bulletin. Included in Appendix C are examples of questions that students must be able to answer to demonstrate *acceptable* or *excellent* achievement.

Students who demonstrate *acceptable* achievement but not *excellent* achievement in Mathematics 30 will receive a final mark between 50% and 79%. Typically, these students have gained new skills and knowledge in mathematics but can anticipate difficulties if they choose to enrol in postsecondary mathematics courses. They have demonstrated mathematical skills and knowledge in the seven content strands of the Mathematics 30 curriculum and an ability to apply a broad range of problem-solving skills to these content strands.

Students who demonstrate *excellent* achievement will receive a final mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in postsecondary mathematics programs; they should be encouraged to pursue careers in which they will use their talents in mathematics.

The specific statements of standards that follow were written primarily to inform Mathematics 30 teachers about the extent to which students must know the Mathematics 30 content and must demonstrate the required skills to pass the examination. The examples provided are by no means exhaustive; they are intended to provide a profile of *acceptable* and *excellent* achievement.

Problem Solving

Students in Mathematics 30 should be able to participate in and contribute towards the problem-solving process for problems within the seven content strands.[1]

The student demonstrating acceptable achievement can:

Given the solution to a problem, analyze the solution for correctness, provide the correct response, and provide possible reasons for the problem solver's errors. For example:

> Menghsha examined the graph of the function $y = 3 \sin \theta$ and determined that the domain of the function was $-1 \leq \theta \leq 1$. Is Menghsha's answer correct? If not, provide the correct answer and explain Menghsha's error.

Given one method of solving a problem, solve it a second way. For example:

> Jillian was asked to find the factors of $P(x) = x^3 - 9x^2 + x - 9$. On her graphing calculator, she graphed the function and determined that the

[1]Italicized comments give an overview of the curriculum statements found in the *Mathematics 30 Course of Studies*.

factors for $P(x)$ were $(x + 3)$, $(x + 1)$, and $(x - 3)$. If Jillian was unable to graph $P(x)$, show another method that Jillian could have used to find its factors.

Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 s, draw a diagram of the situation and create a table of values showing the relationship between the height h of a rider above the ground (the lowest point of the Ferris wheel is 1 m above the ground) and the time t to determine the height of a rider after 6 s.

The student demonstrating excellent achievement can:

Given a problem, solve it for the specific case(s) and then provide a general solution. For example:

Given a 4-sided and a 10-sided polygon, determine the number of diagonals in each. The student can also determine a general statement about the number of diagonals in an n-sided polygon.

Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 s, develop a mathematical model that describes the relationship between the height h of a rider above the lowest point of the Ferris wheel, which is 1 m above the ground, and the time t. The student can provide a full explanation of how his or her model was developed and can suggest alternative ways of developing the model.

Polynomial Functions

Given any integral polynomial function of degree 3 or less, students should be able to determine its zeros, its factors, and its graph, and should be able to describe, orally and in writing, the relationship among its zeros, its factors, and its graph.

The student demonstrating acceptable achievement can:

Given $P(x) = 10x^3 + 51x^2 + 3x - 10$, determine its zeros, its factors, and its graph. The student can also describe the relationship among its zeros, its factors, and its graph.

The student demonstrating excellent achievement can:

Given $P(x) = ax^3 + bx^2 + 3$ and that the remainders are 7 and 10 when divided by $x - 2$ and $x + 1$ respectively, determine the zeros, the factors, and the graph of $P(x)$.

Trigonometric and Circular Functions

Students should be able to solve a first-degree primary trigonometric equation and describe the relationship between its root(s) and the graph of its corresponding function.

Students should also be able to demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

The student demonstrating acceptable achievement can:

Given $y = 2\sin\left(\theta - \frac{1}{2}\right), 0 \leq \theta < 2\pi$, determine its zeros, describe orally and in writing the relationship between its zeros and its corresponding graph, and the effect that 2 and $-\frac{1}{2}$ have on the graph of $y = \sin\theta$.

Given $\frac{\cot\theta}{\tan\theta}$, use fundamental trigonometric identities to simplify the expression and verify this simplification by substituting values for the variable and by comparing their corresponding graphs.

The student demonstrating excellent achievement can:

Given $2 - 2\cos^2\theta = \sin\theta, 0 \le \theta < 2\pi$, determine its zeros and describe orally and in writing the relationship between its zeros and the graphs of $y = 2 - 2\cos^2\theta$ and $y = \sin\theta$.

Statistics

Given a problem whose solution requires the USE of statistics, students should be able to design and administer surveys, collect and organize the results of surveys, draw inferences from surveys INCLUDING bivariate data and yes/no questions, and determine THE CONFIDENCE INTERVALS FOR THE RESULTS OF YES/NO SURVEYS.

Given a set of normally distributed data, students should be able to describe and analyze the data using the characteristics of a normal distribution.

The student demonstrating acceptable achievement can:

Given that a local television store in a community of 100 000 families wishes to find out the number of families that own at least 2 television sets, DESCRIBE AN APPROPRIATE SAMPLE to administer a survey to, design and then administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, predict the number of families that own at least 2 television sets USING 90% CONFIDENCE INTERVALS.

Given that a local television store in a community of 100 000 families wishes to find out WHETHER THE NUMBER OF TELEVISION SETS OWNED BY A FAMILY IS RELATED TO THE NUMBER OF CHILDREN IN THE FAMILY, DESCRIBE AN APPROPRIATE SAMPLE to administer a survey to, design and then administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, DETERMINE whether or not this relationship exists and, if it does, in which direction, AND DETERMINE A LINE OF BEST FIT USING THE MEDIAN FIT METHOD.

Given that the results of a test were normally distributed with a mean of 30 and a standard deviation of 5, and that the passing mark was set at 25, determine the percentage of students who passed the test.

The student demonstrating excellent achievement can:

Given that a local television store in a community of 100 000 families wishes to find out the number of families that own at least 2 television sets, DESCRIBE AN APPROPRIATE SAMPLE to administer a survey to, design and then administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, predict the number of families that own at least 2 television sets USING 90% CONFIDENCE INTERVALS. IN A YES/NO SURVEY, explain orally and in writing the confidence with which inferences to the population were made and DESCRIBE THE MEANING of this confidence level.

Given that a local television store in a community of 100 000 families wishes to find out WHETHER THE NUMBER OF TELEVISION SETS OWNED BY A FAMILY IS RELATED TO THE NUMBER OF CHILDREN IN THE FAMILY, DESCRIBE AN APPROPRIATE SAMPLE to administer a survey to, design and then administer the survey, collect the results, and organize the results to reflect the data collected. Based on the results collected, DETERMINE whether or not this relationship exists and, if it does, in which direction, determine a line of best fit USING THE MEDIAN FIT METHOD and its prediction equation.

Quadratic Relations

Students should be able to describe orally, in writing, and by modeling, the quadratic relation resulting from the intersection of a plane and a conical surface and, from the graph of a quadratic relation, the combination of values for the numerical coefficients of the general quadratic relation that defines each graph and would result in the degenerate quadratic relations. Given the locus defining a conic section and/or the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, they can identify the quadratic relation described.

The student demonstrating acceptable achievement can:

Given $2x^2 + 2y^2 + x - 3y - 25 = 0$, identify the quadratic relation described by this equation.

Given $Ax^2 + Cy^2 + Dx - Ey - 36 = 0$, identify this as a hyperbola when AC < 0.

Given that a quadratic relation is represented by $3x^2 + 4y^2 + 5x + Ey - 36 = 0$, where $B = 0$, describe orally or in writing what happens to the graph of this quadratic relation when 5 is changed to –4 and –36 is changed to –9.

Given a quadratic relation that is described as having an eccentricity of 2, identify this as a hyperbola and describe its locus.

Given that the locus of points such that the sum of the distances between one of the points and two fixed points is constant, identify this locus as an ellipse.

Given a description of the intersection of a plane and a conical surface, identify the conic section formed.

Given that the cutting plane approaches the vertex of the conical surface, describe orally, in writing, and by modelling, the effect on the ellipse.

Given that the eccentricity of the orbit of Halley's Comet, which has a period of 76 years, is 0.96, sketch its graph.

The student demonstrating excellent achievement can:

Describe and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, when one or more of the numerical coefficients change.

Given a description of the intersection of a plane and a conical surface, identify orally or in writing the degenerate parabola formed.

Given the equation of a degenerate quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, describe orally and in writing the quadratic relation formed.

Given the graph and the eccentricity of a quadratic relation, describe orally and in writing the changes to the graph when the eccentricity changes.

Given that the fixed point of an ellipse is moving closer to the centre of the ellipse, describe orally and in writing the effect on the eccentricity.

Given that the eccentricity of any quadratic relation is the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, describe orally and in writing the effect that changing the eccentricity has on the relative positions of the fixed line and the fixed point.

Exponential and Logarithmic Functions

Given an exponential function, students should be able to describe orally and in writing its inverse as the logarithmic function and what it means for this information in the solution of exponential equations.

The student demonstrating acceptable achievement can:

Given $f(x) = 4^{2x}$, sketch its graph, discuss its domain and range, find the zeros of its corresponding equation, and describe orally and in writing the relationship between the zeros of its equation and its graph. The student can write the inverse of $f(x) = 4^{2x}$ in logarithmic form, sketch its graph, discuss its domain and range, determine the zeros of this equation, and describe orally and in writing the relationship between the zeros of its equation and its graph.

The student demonstrating excellent achievement can:

Given the equation $\log_5(x-4) + \log_5(x-2) = 3$, find all the possible values of x, identify the domain, describe orally and in writing the relationship between the zeros of this equation and its graph, and describe orally and in writing the reasons why there are values of x that satisfy the equation but that are not permissible for the function.

Permutations and Combinations

Students should be able to describe orally and in writing the difference between a permutation and a combination and calculate the number of permutations or combinations of ***n*** *things taken* ***r*** *at a time.*

The student demonstrating acceptable achievement can:

Given that there are 10 musicians in the finals of a music competition, decide whether permutations or combinations should be used to calculate the number of ways that first, second, and third prizes can be awarded.

Given the binomial $(x + 2)^5$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing x^4 (2) is obtained, determine the number of terms in the expansion of $(x + 2)^5$, and describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial.

The student demonstrating excellent achievement can:

Given that five people can sit at a round table, decide whether permutations or combinations should be used to determine the number of different orders in which these five can sit at the table if Jack and Jill must sit next to one another. The student can also justify the method of solution.

Given the binomial $(3x - 2y)^7$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $[(3x)^6(2y)]$ is obtained, and determine the number of terms in the expansion of $(3x - 2y)^7$. The student can describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial, how the number of a given term in the expansion of $(3x - 2y)^7$ relates to the exponent of $(2y)$ in that term, and how the coefficients of the terms that are equidistant from the ends of the expansion of $(3x - 2y)^7$ compare in terms of combinations.

Sequences and Series

Given finite arithmetic and geometric sequences, finite arithmetic series, or finite geometric series, students should be able to describe orally and in writing the differences between sequences and series; the differences between finite and infinite; determine the terms of arithmetic and geometric sequences; and determine the sums of arithmetic and geometric series.

The student demonstrating acceptable achievement can:

Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, and determine the sum of a specified number of terms.

Given that a series is defined by $\sum_{n=3}^{6}(-2)^n$, write the terms of the series, determine whether the series is arithmetic or geometric, and determine the sum of the series.

The student demonstrating excellent achievement can:

Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, determine the sum of a specified number of terms, and determine the formula for the sum of n terms.

Given an arithmetic sequence where $t_4 + t_{13} = 99$ and $t_7 = 39$, determine the first term of this sequence.

Student Evaluation would appreciate your feedback on these statements of assessment standards. Please address your concerns or your suggestions for improvement to:

Assistant Director
Mathematics/Sciences
Student Evaluation Branch
Alberta Education
Box 43, 11160 Jasper Avenue
EDMONTON, Alberta
T5K 0L2 FAX: 422-4200

Structure of the 1994 Examinations

Each Mathematics 30 Diploma Examination is designed to reflect the core content outlined in the *Mathematics 30 Course of Studies.* The examination is limited to those expectations that can be measured by a paper and pencil test. The time allotted to write the examination is two and one-half hours. Students may take an additional 0.5 h to complete the examination if needed.

Core Content

The core content for the 1994 Mathematics 30 diploma examinations is emphasized as follows:

Core Content[2]	*Per Cent Emphasis*[3]
Polynomial Functions	12.5
Trigonometric and Circular Functions	18.75
Statistics	18.75
Quadratic Relations	12.5
Exponential and Logarithmic Functions	12.5
Permutations and Combinations	12.5
Sequences and Series	12.5

Design

The design of the 1994 Mathematics 30 diploma examinations is as follows:

Question Format	*Number of Questions*	*Per Cent Emphasis*
Multiple Choice	42	60
Numerical Response	7	10
Written Response	4[4]	30

See Appendix F for a list of directing words that may be used in the written-response section of the examination.

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[2]Core content descriptions have been shortened in this table.
[3]As suggested in the *Mathematics 30/33 Interim Teacher Resource Manual*, Alberta Education Curriculum Branch, 1991, p. 23.
[4]One written-response question will be worth 10% of the examination.

The three mathematical abilities[5] of Procedures, Concepts, and Problem Solving are addressed throughout the examination. Each ability has the following emphasis:

Multiple Choice and Numerical Response	***Per Cent Emphasis***
Procedures	24.5
Concepts	21
Problem Solving	24.5
Written Response	
Procedures, Concepts, Problem Solving	30

Each examination is built as closely as possible to these specifications.

Directions for the 1994 Examinations

The 1994 examinations consists of a multiple-choice section, a numerical-response section, and a written-response section.

In the multiple-choice section, students are to choose the correct or best possible answer from four alternatives.

In the numerical-response section, students are to calculate a numerical answer. As well, students are to record their answer on a separate answer sheet, usually correct to the nearest tenth or nearest hundredth. When the answer students are to record is not a decimal value (e.g., the number of people or the degree of a polynomial), students are asked to determine what "the number of people is __________" or "the degree of this polynomial is _________". If the answer can be a decimal value, then students are asked to record their answer correct to the nearest tenth or nearest hundredth. For instance, numerical-response question 7 on the January 1992 diploma examination asked students to calculate the numerical value of the second term of an arithmetic sequence. Students first had to calculate the common difference and then use that value to determine the value of the second term. Although the value of the second term did result in a whole number, it could have been a decimal value if the value of the common difference was a decimal value. Hence, students were asked to record their answer "correct to the nearest tenth."

The written-response section focuses on students' understanding of the process of solving a problem and encourages students to take risks to arrive at a solution. Students will be rewarded for selecting a problem-solving strategy and for carrying through with the strategy to find a solution. To achieve *excellence*, students must be able to select a strategy, carry it through, and complete the problem. The written-response section of the examination also focuses on students' understanding of mathematical concepts and allows for the most flexibility in gaining an understanding of students' communication and problem-solving abilities in mathematics.

Questions on the written-response section ask students to solve, explain their solution, justify their solution or prove. For a definition of directing words that students may encounter on the diploma examination, see Appendix F.

In scoring the written-response section of the examinations, markers will evaluate students for how well they:

- understand the problem or the mathematical concept,
- correctly use the mathematics,
- use problem-solving strategies and explain their answer and procedures,
- communicate their solutions and mathematical ideas.

Above all, students should be encouraged to try to solve the problem. Even an attempt at a solution could be worth some marks. If students leave the paper blank, markers will not be able to award any marks. Please share the information found in Appendix G and H with your students. It may

[5]An explanation of mathematical abilities is given in Appendix D.

help them preparing for and writing a diploma examination.

Mathematics as Communication

In keeping with the expectations listed in the *Mathematics 30 Course of Studies*, the 1994 examinations will reflect mathematics as communication. The program of studies includes communication in the problem-solving expectations: "Students will be expected . . . to read the problem thoroughly; identify and clarify key components; restate the problem, using familiar terms . . . ask relevant questions . . . document the solution process . . . and explain the solution in oral or written form . . ." (From *Mathematics 30 Course of Studies*, pp. 6-7).

These expectations are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9–12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can
>
> - reflect upon and clarify their thinking about mathematical ideas and relationships;
> - formulate mathematical definitions and express generalizations discovered through investigations;
> - express mathematical ideas orally and in writing;
> - read written presentations of mathematics with understanding;
> - ask clarifying and extending questions related to mathematics they have read or heard about;
> - appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.
>
> Focus: All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas.
>
> (From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 140)

The National Council of Teachers of Mathematics describes the evaluation of mathematics as communication in the following manner:

> The assessment of students' ability to communicate mathematics should provide evidence that they can
>
> - express mathematical ideas by speaking, writing, demonstrating, and depicting them visually;
> - understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms;
> - use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations.
>
> (From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 214)

Beside writing to communicate results, the accuracy of and logic in students' mathematical statements also reflects mathematics as communication. Hence, students will continue to be expected to demonstrate logical and meaningful communication on the diploma examinations.

Mathematics as Problem Solving

In keeping with the expectations identified in the *Mathematics 30 Course of Studies*, the 1994 examinations will reflect mathematics as problem solving. Problem solving is integrated throughout the content areas in the curriculum. A set of specific problem-solving learner expectations is identified before the specific content learner expectations.

The expectations contained in the program of studies are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9–12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can
>
> - use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;

- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

Focus: In grades 9–12, the problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the student's approach to doing mathematics, regardless of the topic at hand. From this perspective, problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics as identified in later standards is both constructed and reinforced.

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 137)

Evaluating Communication and Problem Solving

The open-ended question is a way in which to examine mathematics as communication and mathematics as problem solving. An open-ended question allows students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

Alberta Education recommends the following two documents, which examine the open-ended question in further detail:

> *Assessment Alternatives in Mathematics*: An overview of assessment techniques that promote learning.
>
> *A Question of Thinking*: A First Look at Students' Performance on Open-ended Questions in Mathematics.

Both these documents are available through:

> California State Department of Education
> Bureau of Publications, Sales Unit
> P. O. Box 271
> Sacramento, CA 95802-0271

The following document was recently published by the National Council of Teachers of Mathematics. It provides some good practical suggestions for the assessment of problem solving and communication on a regular basis:

> *Mathematics Assessment*: Myths, Models, Good Questions, and Practical Suggestions.

This document is available through:

> National Council of Teachers of Mathematics
> 1906 Association Drive
> Reston, VA 22091

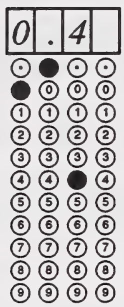
The Answer Sheet

There will continue to be a common answer sheet for the machine-scored part of the 1994 series of diploma examinations for mathematics, chemistry, and physics. All three subjects will use common instructions and a common form for the numerical-response questions. The format of the answer sheet allows students to place the decimal point in an appropriate position. In **all** cases, students will be required to fill in the answer beginning at the left field and leave any unused fields blank. See the following page for examples of filling in the answer sheet and see Appendix E for an explanation of significant digits and rounding.

The following examples illustrate the use of the answer sheet.

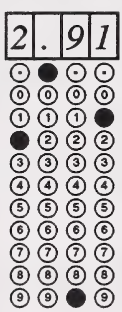
Example 1: If $\csc\theta = 2.6$, $\frac{\pi}{2} < \theta < \pi$, then the value of $\sin\theta$ correct to the nearest tenth is ______.

Value: 0.3846...
Value to be recorded: 0.4



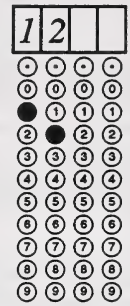
Example 2: $P(x, 1.54)$ is a point on the graph of $y = \log_2(x)$. Correct to the nearest hundredth, the value of x is ______.

Value: 2.9079...
Value to be recorded: 2.91



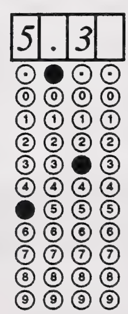
Example 3: If $x^{10} - 8x + 3$ is divided by $x + 1$, then the remainder is ______.

Value: 12
Value to be recorded: 12



Example 4: A university biology class consists of 200 students: The marks of the final examination are normally distributed with a mean mark of 45.0 and a standard deviation of 5.3. The professor adjusts the marks by adding 5.0 to each grade. Correct to the nearest tenth, the standard deviation of the adjusted marks is ______.

Value: 5.3
Value to be recorded: 5.3



The Formula Sheet

There will continue to be a formula sheet, a *z*-score page, and 90% Box Plots provided in all 1994 examinations.

A proposed formula sheet for the 1994 examinations was published in the MCATA Newsletter in the spring of 1993, and was shared with many school districts in the province and with markers at the July 1993 marking session. Based on comments that were received, the formula sheet was revised in the following ways:

Added:

- definition of both the quadratic formula and the distance formula
- $S_n = \dfrac{rt_n - a}{r - 1}$, $r \neq 1$ in **Sequences and Series**
- logarithmic laws from the *Mathematics 30 Course of Studies*

Removed:

- $P(x) = D(x)Q(x) + R$ from **Polynomial Functions**
- $y = mx + b$ from **Statistics**
- $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ from **Quadratic Relations**
- "where F = focus, D = directrix, and P = point on the conic" from **Quadratic Relations**
- $n! = n(n-1)(n-2)\ldots(3)(2)(1)$ from **Permutations and Combinations**

Moved:

- $z = \dfrac{x - \mu}{\sigma}$ from **Statistics** on Formula Sheet to the top of the *z*-score page
- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ from **Polynomial Functions** to the top of the page
- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ from **Quadratic Relations** to the top of the page

The contents of the 1994 Formula Sheet and *z*-score page are reproduced on the following two pages of this Bulletin. There are no changes to the 90% Box Plots.

Mathematics 30 Formula Sheet

The following information may be useful in writing this examination.

- The roots of the quadratic equation $ax^2 + bx + c = 0$ are
  $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- The distance between two points (x_1, y_1) and (x_2, y_2) is
  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Quadratic Relations

- $e = \dfrac{|PF|}{|PD|}$

Trigonometry

- arc length $a = r\theta$
- $\sin^2 A + \cos^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$
- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\csc A = \dfrac{1}{\sin A}$
- $\sec A = \dfrac{1}{\cos A}$
- $\cot A = \dfrac{\cos A}{\sin A}$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Permutations and Combinations

- ${}_nP_r = \dfrac{n!}{(n-r)!}$
- ${}_nC_r = \dfrac{n!}{r!(n-r)!}$
- In the expansion of $(x + y)^n$, the general term is $t_{k+1} = {}_nC_k\, x^{n-k} y^k$.

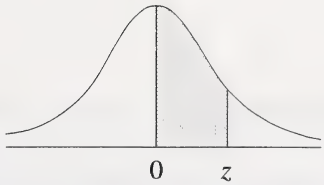
Sequences and Series

- $t_n = a + (n - 1)d$
- $S_n = \dfrac{n[2a + (n - 1)d]}{2}$
- $S_n = n\left(\dfrac{a + t_n}{2}\right)$
- $t_n = ar^{n-1}$
- $S_n = \dfrac{a(r^n - 1)}{r - 1}, r \neq 1$
- $S_n = \dfrac{rt_n - a}{r - 1}, r \neq 1$

Exponential and Logarithmic Functions

- $\log_a mn = \log_a m + \log_a n$
- $\log_a \dfrac{m}{n} = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$

$$z = \frac{x - \mu}{\sigma}$$



Areas under the Standard Normal Curve

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Appendix A

Policy: Use of Calculators on Alberta Education Diploma Examinations

Background

The knowledge, skills and attitudes relevant to technology and its uses are being incorporated into courses and programs of study wherever appropriate. Students are expected to learn the advantages and limitations of technological developments and their impact upon society. The ability to use technology helps students understand and appreciate the process of technological change, gives added depth to programs, and provides the basis for the development of skills and understanding. These expectations are reflected in the diploma examinations. Since the data provided for writing diploma examinations in mathematics, chemistry, and physics does not include information such as logarithms and trigonometric functions, students will need to use scientific calculators for these exams.

Definition

This policy considers a scientific calculator to be a hand-held device designed primarily for mathematical computations. Included in this definition are those scientific calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features.

Policy

To ensure compatibility with provincial *Programs of Study* and equity and fairness for all students, Alberta Education expects students to use scientific calculators, as defined above, when they are writing diploma examinations in mathematics, chemistry, and physics. Examinations are constructed to ensure that the use of particular models of calculators neither advantages nor disadvantages individual students.

Procedures

1. At the beginning of a course, teachers must advise students of the calculators that may be used when they are writing mathematics, chemistry, and physics diploma examinations.

2. In preparation for calculator failure, students may bring extra calculators and batteries into the examination room.

3. During exams, supervising teachers must ensure that
   a. all calculators fall within the definition provided with this policy,
   b. all calculators operate in silent mode,
   c. students do not share calculators,
   d. students do not bring external devices to support calculators into the examination room. Such devices include manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, or any annotations outlining operational procedures for scientific calculators.

Appendix B

Examination Rules, Grade 12 Diploma Examinations

1. **Admittance to the Examination Room**
   Students must not enter or leave the examination room without the consent of the supervising teacher.

2. **Student Identification**
   Students must present identification that includes their signature and photograph. One of the following documents is acceptable: driver's licence, passport, or student identification card. Students must not write an examination under a false identity or knowingly provide false information on an application form.

3. **Identification on Examinations**
   Students must not write their names or the name of their school anywhere in or on the examination booklet other than on the back cover.

4. **Time**
   Students must write an examination during the specified time and may not hand in a paper until at least one hour of the examination time has elapsed. Students who arrive more than one hour after an examination has started will not be allowed to write the examination. Students who arrive late but within the first hour of an examination sitting may be allowed to write only at the discretion of the supervising teacher.

5. **Discussion**
   Students must not discuss the examination with the supervising teacher unless the examination is incomplete or illegible. Students must not talk, whisper, or exchange signs with one another.

6. **Answer Sheets**
   Students must use an HB pencil to record their answers on the machine-scorable answer sheets.

7. **Written Responses**
   All work for the written-response sections of the diploma examinations must be done in the examination booklet. Students are expected to write their revised work in blue or black ink for English 30, English 33, Français 30, Social Studies 30, and Biology 30.

8. **Material Exchanges**
   Students must not copy from other students or exchange material. Notes in any form–including those on papers, in books, or stored in electronic devices–must not be brought into the examination room. Calculator programs designed to perform mathematical computations or those designed to assist students in graphing are not classified as notes

9. **Material Allowed**
   English 30, English 33: Students may use a dictionary and a thesaurus for Part A only. Electronic devices are not allowed for either part.
   Français 30: Students may use a dictionary, a thesaurus, and a book of verb forms for Partie A only. Electronic devices are not allowed for either part.
   Social Studies 30, Biology 30: Students may not use electronic devices.
   Mathematics 30: Tear-out data pages are provided in the examination booklet. Students may use scientific calculators (see Calculator Policy, General Information Bulletin) but must not share them.
   Chemistry 30, Physics 30: A separate data booklet is provided for each of these examinations. Students may use scientific calculators (see Calculator Policy, General Information Bulletin) but must not share them.

   Students are expected to provide their own writing materials, including pens and HB pencils, calculators, or other necessary instruments. Tear-out pages for rough work are provided in each biology, chemistry, mathematics, and physics examination booklet.

10. **Translation Dictionaries**
    Students are not allowed to use translation dictionaries in any subject. Exchange students must satisfy the same requirements as other students.

Appendix C

Mathematics 30 Curriculum Standards

The Curriculum Standards provided in this appendix are intended to clarify the *Mathematics 30 Course of Studies* statements. Included are examples of questions that students must be able to do to demonstrate *acceptable* or *excellent* achievement. For a definition of *acceptable* and *excellent* achievement, see page 3 of this Bulletin.

Problem Solving

Students in Mathematics 30 can participate in and contribute towards the problem-solving process for problems within the seven content strands.[1]

Polynomial Functions

Given any integral polynomial function of degree 3 or less, students can determine its zeros, its factors, and its graphs and can describe, in writing, the relationship among its zeros, its factors, and its graphs.

The student demonstrating acceptable achievement can:

- recognize and give examples of polynomial function of different degrees;
- generate the graph of any integral polynomial function with the use of graphing calculators or graphing utility packages;
- use the Remainder Theorem to evaluate a third-degree integral polynomial function for rational values of the variable and to understand how this can be used to find factors of the polynomial function;
- factor and find the zeros for an integral polynomial function in standard form, degree 3 or less, in which all zeros are rational;
- find approximations for all the real zeros of integral polynomial functions using graphing calculators or computers;
- derive an equation of an integral third-degree polynomial function given its rational zeros;
- recognize the general shape of graphs of integral polynomial functions of degree 4 or less where the multiplicity of zeros is one, two, or three;
- identify the potential rational zeros of an integral polynomial function;
- determine the minimum degree of a polynomial function by using the multiplicities of its zeros;
- participate in and contribute toward the problem-solving process for problems that can be represented by polynomial functions studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- use the Remainder Theorem when either the factor or the original polynomial contains unknown coefficients;
- explain the relationships between the graphs of different polynomial functions and their zeros;
- derive an equation for an integral polynomial function given its zeros and any other information that will uniquely define it;
- use the Remainder Theorem to evaluate integral polynomial function beyond the third degree for rational values of the variable and understand how this can be used to find factors of the polynomial function;
- recognize the general shape of graphs of integral polynomial functions of degree n where the multiplicity of zeros is greater than two;
- complete the solution to problems that can be represented by polynomial functions studied in Mathematics 30.

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[1] Italicized comments give an overview of the curriculum statements found in the *Mathematics 30 Course of Studies.*

The student demonstrating acceptable achievement can do the following types of questions:

1. One factor of $10x^3 + 51x^2 + 3x - 10$ is $x + 5$. The other two factors are

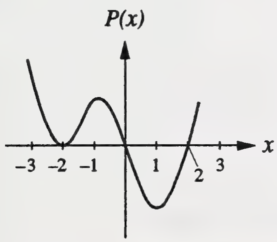
* **A.** $2x + 1$ and $5x - 2$
B. $2x - 1$ and $5x + 2$
C. $2x + 5$ and $5x - 1$
D. $2x - 5$ and $5x - 1$

2. For an integral polynomial function $P(x)$, $P(5) = 0$ and $P(-2) = 0$. One factor of this polynomial is

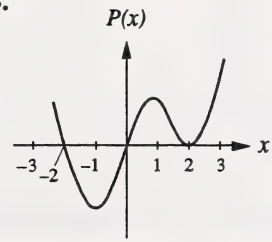
A. $x - 2$
B. $x + 5$
* **C.** $x^2 - 3x - 10$
D. $x^2 + 3x - 10$

3. The sketch that illustrates the graph of $P(x) = ax(x + 2)(x - 2)^2$, where $a > 0$, is

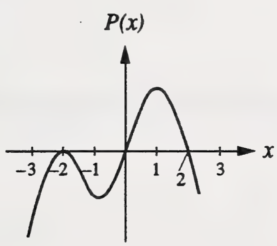
A.



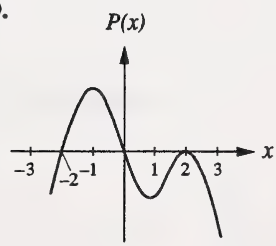
* **B.**



C.



D.



The student demonstrating excellent achievement can also identify the sketch that illustrates the graph of $P(x) = -ax(x + 2)(x - 2)^2$, where $a > 0$.

4. When $5x^3 - 7x^2 + 2x + 1$ is divided by $x - 3$, the remainder correct to the nearest tenth is _[79.0]*.

---

*Correct answer.

The student demonstrating excellent achievement can do questions such as:

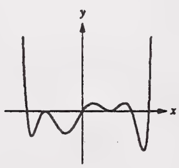
1. When $ax^2 + bx + 5$ is divided by $x - 2$, the remainder is 7, and when divided by $x + 1$, the remainder is 10. The value of b is

   A. 3
   B. 2
   C. –2
 * **D.** –3

2. If –1 and –2 are x-intercepts of the graph of $y = x^3 + ax^2 - x + b$, then the values of a and b respectively are

   A. 2 and 2
 * **B.** 2 and –2
   C. –2 and 2
   D. –2 and –2

3. A calculator display showed the following graph.



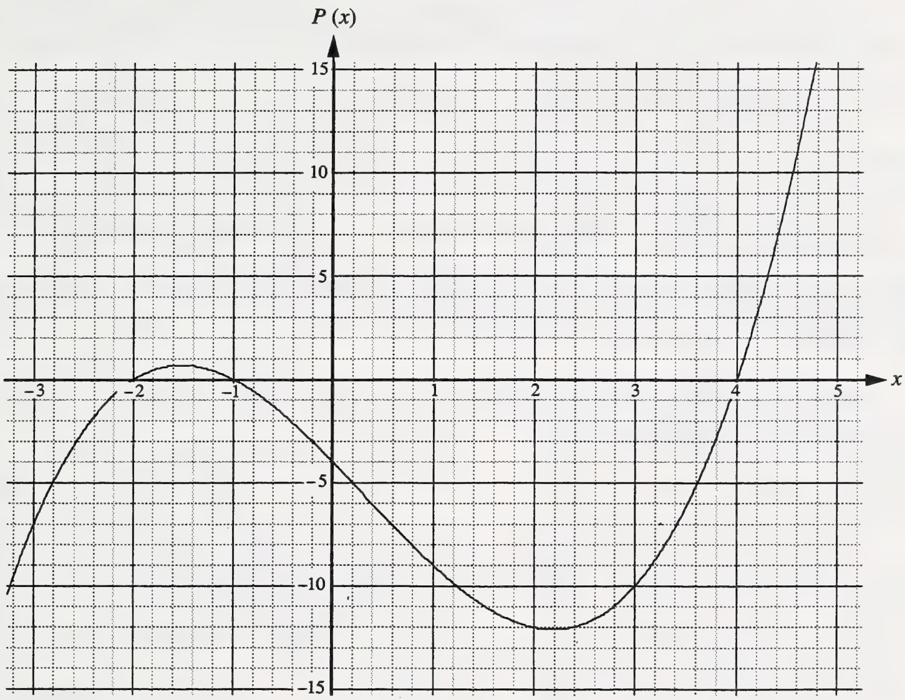
   The lowest degree of the polynomial represented by the graph is

   A. 5
   B. 6
   C. 7
 * **D.** 8

4. If $2x^3 + kx^2 - mx - 5$ is divided by $x - 3$, the remainder is 6. The equation relating k and m is

   A. $9k - 3m = -49$
 * **B.** $9k - 3m = -43$
   C. $9k + 3m = 59$
   D. $9k + 3m = 65$

5. The graph of $P(x) = a(x - p)(x - q)(x - r)$, as represented below, is displayed on a computer screen. Pat's assignment is to find the value of a. From the graph, Pat notes that the x-intercepts are -1, -2, and 4.



Pat finds that the value of a is

A. $\frac{3}{4}$

* B. $\frac{1}{2}$

C. -4

D. -10

6. If $x - c$ is a factor of $6x^3 + 3cx^2 - c^2x - 27$, then the value of c correct to the nearest tenth is__[1.5]*.

The student demonstrating acceptable achievement can successfully *complete part a* of the following three-part question, whereas the student demonstrating excellent achievement can successfully *complete all three parts* of this three-part question.

7. The graph of a third-degree polynomial function touches the x-axis at $(1, 0)$ and crosses the x-axis at $(-2, 0)$. Express in factored form

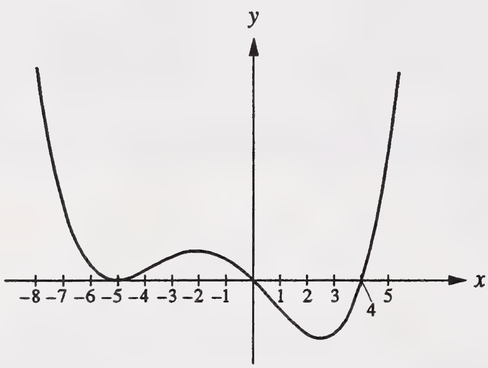
   a. an equation for such a polynomial function

   b. the equation of the polynomial function if the y-intercept of its graph is -6

   c. the equation of the polynomial function if its graph passes through $(2, 8)$

The student demonstrating acceptable achievement can successfully *complete parts a and b* of the following three-part question, whereas the student demonstrating excellent achievement can successfully *complete all three parts*.

8. This is a partial sketch of the graph of a polynomial function with domain $-8 \le x \le 5$. There are no other x-intercepts.



   a. What are the zeros of this polynomial?

   b. What is the lowest possible degree of this polynomial?

   c. What other degrees could this polynomial be? Explain your answer.

Trigonometric and Circular Functions

Students can solve a first-degree primary trigonometric equation and describe the relationship between its root(s) and the graph of its corresponding function.

Students can also demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

The student demonstrating acceptable achievement can:

- convert angle measurements between degree and radian measure;
- given any two of the following measurements—the radian measure of the central angle, the radius, or the length of an arc—determine the unknown measurement;
- verify[3] the fundamental trigonometric identities;
- solve first-degree trigonometric equations on the domain $0 \leq \theta < 2\pi$ in radians and $0° \leq \theta < 360°$;
- simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities;
- generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages;
- explain the effect of each parameter a, b, c, and d on the graph of the $y = a\sin[b(\theta + c)] + d$ and $y = a\cos[b(\theta + c)] + d$ functions;
- state the domain and range of $y = \sin\theta$, $y = \cos\theta$, and $y = \tan\theta$;
- describe, orally and in writing, the relationship between the root(s) of a first-degree trigonometric equation and the graph of its corresponding function;
- participate in and contribute toward the problem-solving process for problems that can be represented by trigonometric functions studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- prove[4] trigonometric identities;
- explain, orally and in writing, the combined effects of the parameters a, b, c, and d in the trigonometric functions $y = a\sin[b(\theta + c)] + d$ and $y = a\cos[b(\theta + c)] + d$, on the functions' domain and range;
- solve first- and second-degree trigonometric equations including double and half angles on the domain $0 \leq \theta < 2\pi$ and $0° \leq \theta < 360°$;
- describe, orally and in writing, the relationship between the root(s) of a trigonometric equation and the graph of its corresponding function;
- complete the solution to problems that can be represented by trigonometric functions studied in Mathematics 30.

[3] For a definition of verify, see the *Mathematics 30 Course of Studies*, p. 6.

[4] For a definition of prove, see the *Mathematics 30 Course of Studies*, p. 6.

The student demonstrating acceptable achievement can do the following types of questions:

1. Correct to the nearest tenth of a radian, an angle of 105° is

* A. 1.8 rad
B. 2.4 rad
C. 4.0 rad
D. 5.4 rad

2. The expression $\frac{\cot\theta}{\tan\theta}$ is equivalent to

A. $\frac{\cos\theta}{\sin\theta}$

B. $\frac{\sin\theta}{\cos\theta}$

C. $\frac{\sin^2\theta}{\cos^2\theta}$

* D. $\frac{\cos^2\theta}{\sin^2\theta}$

3. If the graph of $y = \sin\theta$ undergoes a phase shift of $\frac{\pi}{2}$ radians to the right and an amplitude increase to π, then the equation of the resulting graph is

A. $y = \frac{\pi}{2}\sin(\theta + \pi)$

B. $y = \frac{\pi}{2}\sin(\theta - \pi)$

* C. $y = \pi\sin\left(\theta - \frac{\pi}{2}\right)$

D. $y = \pi\sin\left(\theta + \frac{\pi}{2}\right)$

4. The expression $\frac{1-\sin^2\theta}{\sin^2\theta}$, where $\theta \neq n\pi, n \in \mathrm{I}$, is equivalent to

* A. $\cot^2\theta$
B. $\tan^2\theta$
C. 1
D. 0

5. If $\sin\theta = \frac{5}{13}, \frac{\pi}{2} < \theta < \pi$, then the value of $\csc\theta$ correct to the nearest tenth is ____[2.6]*.

The student demonstrating excellent achievement can also do questions such as:

1. The expression $\dfrac{\sec\theta\sin\theta}{\csc\theta\cos\theta}$ is equivalent to

* **A.** $\tan^2\theta$
B. $\cot^2\theta$
C. $\sin\theta\cos\theta$
D. $\sin^2\theta\cos^2\theta$

2. If $2 - 2\cos^2\theta = \sin\theta, 0 \le \theta < 2\pi$, then all possible values of θ are

A. $0, \frac{\pi}{2}, \pi$

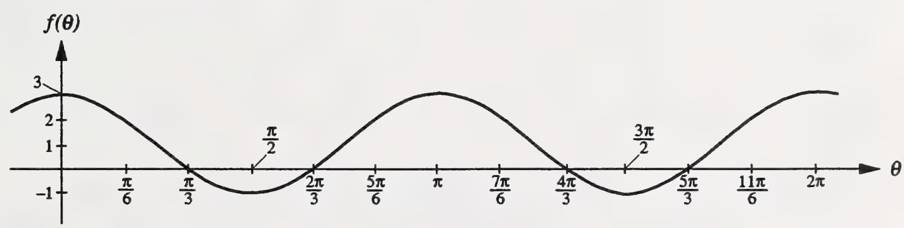
* **B.** $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

C. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

D. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

3. If the solutions to $A\sin^2\theta - B\sin\theta + 1 = 0, 0° < \theta \le 90°$, are 30° and 90°, then the value of B correct to the nearest tenth is ____[3.0]*.

4. The graph of $f(\theta) = 2\cos(2\theta) + 1$, as represented below, is displayed on a computer screen.



Kelly is asked to find all the values of θ that satisfy the equation $2\cos(2\theta) = -1$, $0 \le \theta \le 2\pi$. Kelly finds that all the values of θ that satisfy this equation are

A. $\frac{\pi}{2}, \frac{3\pi}{2}$

B. $\frac{2\pi}{3}, \frac{4\pi}{3}$

C. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

* **D.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

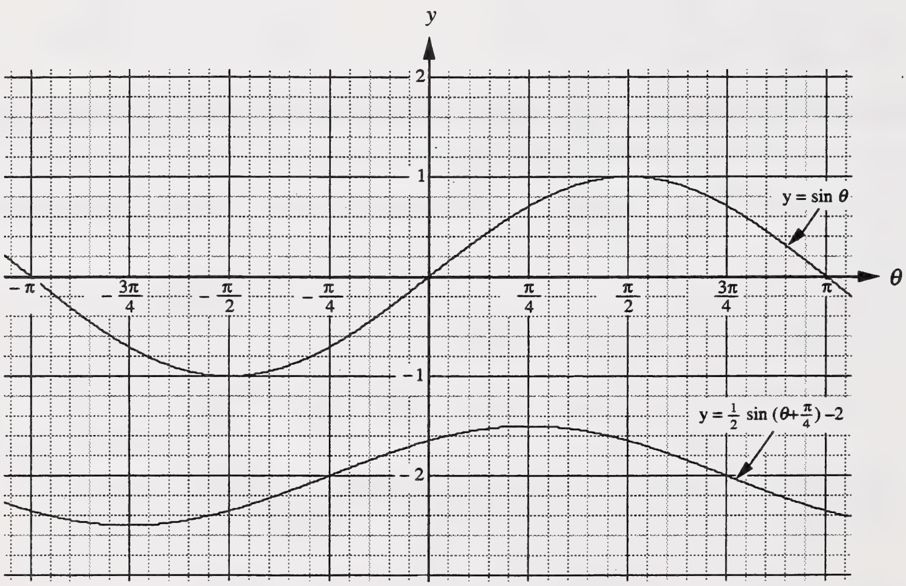
The student demonstrating acceptable achievement can successfully *complete the initial substitutions for csc θ and cot θ* in this written-response question, whereas the student demonstrating excellent achievement can successfully *complete* the proof.

5. Prove that $(1 + \cos\theta)(\csc\theta - \cot\theta) = \sin\theta$, where $\theta \neq n\pi$, $n \in I$.

   SHOW CLEARLY ALL SUBSTITUTIONS AND PROCEDURES.

The student demonstrating acceptable achievement can successfully *describe the individual effects of the parameters* $\frac{1}{2}$, $+\frac{\pi}{4}$, and -2 on the graph of $y = \sin\theta$ in this written-response question, whereas the student demonstrating excellent achievement can also *describe the effects of these parameters on the domain or range* of the function.

6. You are helping your friend analyze the graphs of trigonometric functions. Your friend wants to know the effects of the parameters a, b, c, and d in $y = a \sin b(\theta + c) + d$. You start by graphing $y = \sin\theta$ and $y = \frac{1}{2}\sin\left(\theta + \frac{\pi}{4}\right) - 2$ as shown below.



Describe the effects of the parameters $\frac{1}{2}$, $+\frac{\pi}{4}$, and -2 on the graph of $y = \sin\theta$.

DRAFT

Statistics

Given a problem whose solution requires the USE of statistics, students should be able to design and administer surveys, collect and organize the results of surveys, draw inferences from surveys INCLUDING bivariate data and yes/no questions, and determine THE CONFIDENCE INTERVALS FOR THE RESULTS OF YES/NO SURVEYS.

Students can also describe and analyze data using the characteristics of a normal distribution.

The student demonstrating acceptable achievement can:

- collect and plot bivariate data;
- design and administer surveys, collect and organize results, and draw inferences from surveys INCLUDING bivariate data and yes/no questions;
- assess the strengths, weaknesses, and biases of samples;
- recognize and describe the apparent correlation between the variables of a bivariate distribution from a scatter plot;
- plot a line of best fit on a scatter plot using the median fit method;
- use charts of 90% box plots to DETERMINE the confidence interval OF THE PROPORTION OF YESES IN THE POPULATION;
- interpret the mean and standard deviation of a set of normally distributed data;
- apply the standard normal curve and the z-scores of data that are normally distributed;
- participate in and contribute toward the problem-solving process for problems that require the analysis of statistics studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- describe, orally and in writing, what is meant by the confidence interval FOR THE PROPORTION OF YESES IN THE POPULATION;
- develop and use prediction equations for a line of best fit to make inferences for populations;
- complete the solution to problems that require the analysis of statistics studied in Mathematics 30.

The student demonstrating acceptable achievement can do the following types of questions:

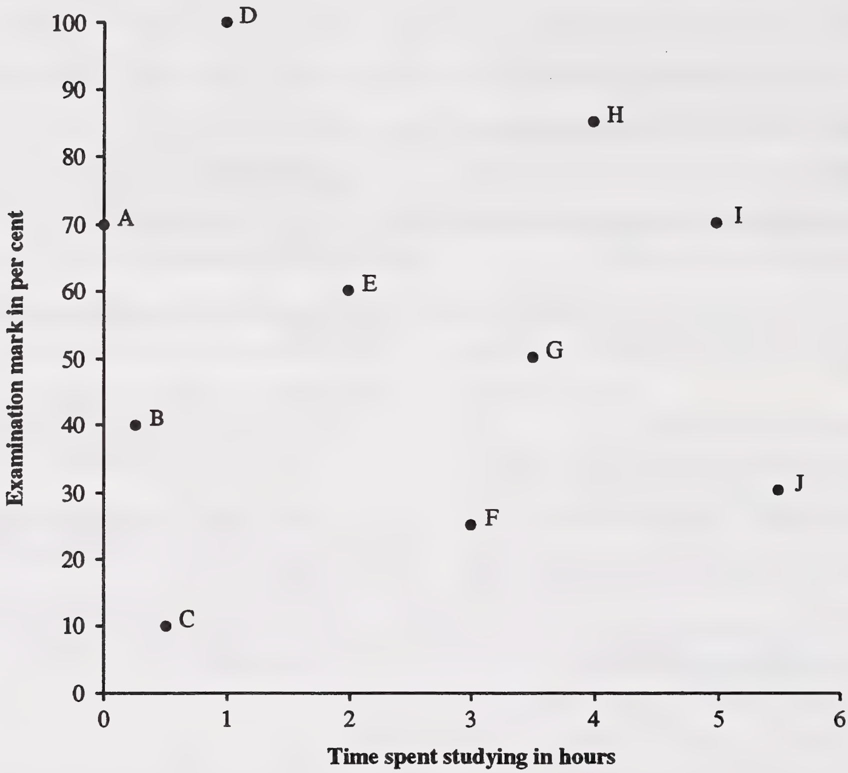
1. The results of an examination are found to be normally distributed with a standard deviation of 8.3. Michelle's score of 75 on this examination corresponds to a z-score of 1.35. The mean for this examination correct to the nearest tenth, is ___[63.8]*.

2. A mark of 73 on an examination translates to a z-score of 1.6. If the mean is 64, then the standard deviation correct to the nearest tenth is ____[5.6]*.

3. The length of the confidence intervals increases as the

* **A.** sample size decreases
B. sample size increases
C. length of the questionnaire increases
D. length of the questionnaire decreases

4. The examination mark received by each of 10 students and the amount of time each student spent studying are shown on the scatter plot below. The students are represented by the letters A to J.



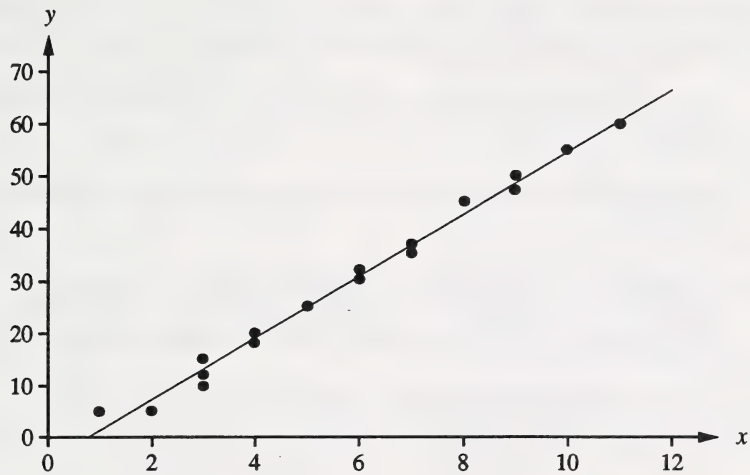
Using the information obtained from the scatter plot, one could say that the time spent studying

* **A.** correlates not obviously with the examination mark received
B. correlates positively with the examination mark received
C. correlates strongly and positively with the examination mark received
D. correlates negatively with the examination mark received

D R A F T

The student demonstrating excellent achievement can also do questions such as:

1. Data and the corresponding line of best for for $1 \leq x \leq 12$ are shown on the following scatter plot.



The equation of the line of best fit (or the prediction equation) for these data is most likely to be

A. $y = 0.9x - 5$
B. $y = 0.9x + 1$
* **C.** $y = 6x - 5$
D. $y = 6x + 1$

The student demonstrating acceptable achievement can *successfully complete parts a and b* in this written-response question. The student demonstrating excellent achievement can *successfully complete all three parts* in this written-response question.

2. Forty bus commuters were asked if they believed their bus service was adequate. Sixteen of the 40 commuters answered "yes".

a. Using the "90% Box Plots from Samples of Size 40" tear-out page near the end of the booklet, determine the 90% confidence interval for the percentage of yeses in the population.

b. Describe what is meant by this 90% confidence interval.

c. How would this confidence interval change if the sample size increased?

The student demonstrating acceptable achievement can successfully *complete part a and identify one factor in part b* in this written-response question. The student demonstrating excellent achievement can complete part a, *can identify one factor in part b and can clearly explain how this factor affects the results of the survey* in this written-response question.

3. A well-constructed survey of Alberta high school students shows that 75% believe the penalties for drinking and driving should be increased.

   a. Assume that the same proportion of 75% was obtained with a sample of size 40 and a sample of size 100.

      Determine the approximate 90% confidence intervals for the survey with these two samples.

   b. A similar survey, with the same sampling methods (survey design) and with the same questions, was repeated in a large Alberta high school. Only 65% of the students at that high school believe the penalties for drinking and driving should be increased.

      From a **statistical perspective**, identify one factor that might have contributed to the results of the survey in this large Alberta high school being different from the results of the survey of Alberta high school students. Explain how this factor affects the results.

Quadratic Relations

Students can describe the conditions that generate quadratic relation sections.

The student demonstrating acceptable achievement can:

- describe orally, in writing, and by modeling, the intersection of a plane and a conical surface that would result in a hyperbola, an ellipse, a parabola, and a circle;
- describe, orally, in writing, and by modeling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate ellipse and hyperbola;
- recognize each quadratic relation, given the locus definition;
- describe, orally and in writing, the quadratic relation, given the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed horizontal or vertical line;
- generate the graph of quadratic relations with the use of graphing calculators or graphing utility packages;
- describe, orally and in writing, and identify the quadratic relation defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$;
- describe, orally and in writing, and identify the effect on the value of b in the equation of a quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, when the graph of the quadratic relation is rotated from its position when $B = 0$;
- describe, orally and in writing, and identify the quadratic relation formed when given the value of the eccentricity;
- describe, orally and in writing, and identify the eccentricity when given the quadratic relation;
- describe, orally and in writing, and identify the quadratic relation formed when given the locus definition;

- identify and graph the quadratic relation when given a point on the quadratic relation, a fixed point, and the eccentricity;
- calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a point on the quadratic relation;
- identify and graph the quadratic relation when given the eccentricity, a fixed point, and a fixed horizontal or vertical line;
- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when TWO of the numerical coefficients change;
- participate in and contribute toward the problem-solving process for problems that require the analysis of quadratic relations studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- describe, orally and in writing, the combination of values for the numerical coefficients of the general quadratic relation that would result in the degenerate conics;
- use the locus definition to verify the equation of each conic section;
- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when TWO or more of the numerical coefficients change;
- describe, orally and in writing, and identify the changes in the graph of a quadratic relation when the eccentricity changes;
- describe orally, in writing, and by modeling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate parabola;
- complete the solution to problems that require the analysis of quadratic relations studied in Mathematics 30.

The student demonstrating acceptable achievement can do the following types of questions:

1. The quadratic relation represented by $2x^2 + x - 3y - 25 = 0$ is

   A. a circle
   * **B.** a parabola
   C. an ellipse
   D. a hyperbola

2. A quadratic relation is represented by $3x^2 + 4y^2 + 5x + Ey - 36 = 0$. Predict and then describe the changes that will happen to the graph when +5 is changed to –4 and –36 is changed to –9.

3. A quadratic relation is described as having an eccentricity of 2. This quadratic relation is

   A. a circle
   B. a parabola
   C. an ellipse
   * **D.** a hyperbola

4. The orbit of a comet has an eccentricity of 1.3. Describe the path that this orbit is following.

5. A conical surface is intersected by a plane that is parallel to its generator. The shape of the conic section formed is

   A. a circle
 * B. a parabola
   C. an ellipse
   D. a hyperbola

6. An object moves along a path such that the sum of the distances from two fixed points is constant. The path of the object can be described as

   A. a circle
 * B. an ellipse
   C. a parabola
   D. a hyperbola

7. Find an equation of a horizontal or vertical directrix that corresponds to a quadratic relation with eccentricity of $\frac{1}{2}$, a point on the quadratic relation of (–2, 3) and a focus of (–4, 0). [4 possible solutions: $y = 3 \pm 2\sqrt{13}$ OR $x = 2 \pm 2\sqrt{13}$]*

8. Halley's Comet has a period of 76 years; that is, Halley's Comet is seen once every 76 years. The orbit of Halley's Comet has an eccentricity of 0.96. Sketch the graph of the orbit.

9. A plane intersects a conical surface. The plane is perpendicular to its axis and passes through the vertex. The locus produced by the intersection of the conical surface and the plane is

   A. a line
 * B. a point
   C. a circle
   D. an ellipse

10. Using a computer, Michele and Robin graphed a quadratic relation defined by $25x^2 + 16y^2 + 30x - 400 = 0$. Now they wish to graph a circle. Which value should Michele and Robin increase to graph the circle?

   A. The coefficient of x^2
 * B. The coefficient of y^2
   C. The coefficient of x
   D. The constant term

11. Describe the effect on an ellipse as the cutting plane approaches the vertex of the conical surface.

The student demonstrating excellent achievement can also do questions such as:

1. A quadratic relation is represented by $Ax^2 + Cy^2 + 3x + Ey - 36 = 0$. Predict and then describe what happens to the graph of the quadratic relation when 3 is changed to –4 and –36 is changed to –9.

2. The equation $Dx + F = 0$ is the **complete** equation of a degenerate

   A. circle
   * **B.** parabola
   C. ellipse
   D. hyperbola

3. As the focus moves closer to the centre of an ellipse, describe the effect on the eccentricity.

4. Given a fixed line and a fixed point, describe the location of a point on a quadratic relation as the eccentricity changes.

5. Yui Lin is sketching the graph of a conic. First she plots a point P at (6, 0). Then she plots two other points on the conic, Q (9, 4) and R (9, –4). What additional information does Yui Lin need in order to know whether the conic will be a hyperbola or a parabola?

   A. The line $y = 0$ is the axis of symmetry
   B. The line $x = 6$ is tangent to the curve
   * **C.** The domain of the relation
   D. The range of the relation

6. The equation $Ax^2 - Cy^2 = 0$ is the equation of a degenerate

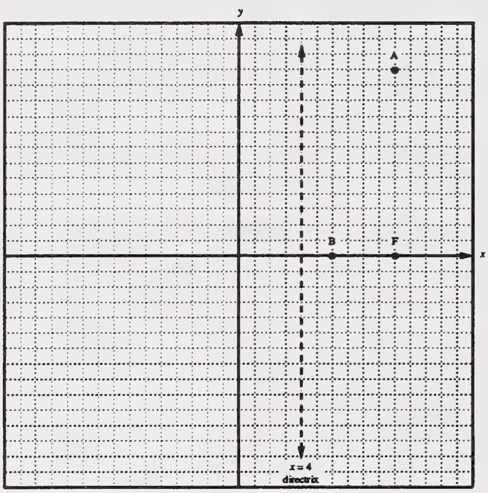
   A. circle
   B. parabola
   C. ellipse
   * **D.** hyperbola

The student demonstrating acceptable achievement can *completely draw the quadratic relation* described in this written-response question.

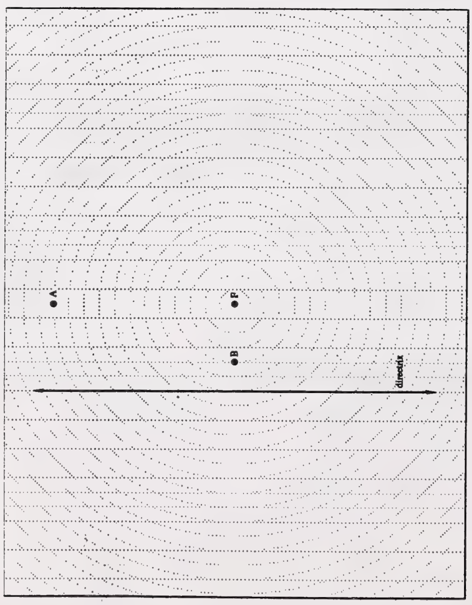
7. Before class ended, Gene and Alex started drawing the graph of the same conic. Both drew the fixed line (directrix) and plotted a fixed point (focus). Just as the bell rang to end the class, Gene and Alex plotted two other points, A and B, on the conic. Below, on the Cartesian plane, is Gene's graph and on the circle line grid, is Alex's graph.

   Complete either Gene's or Alex's graph. Show how you decided which conic the students were drawing.

Gene's Graph



Alex's Graph



DRAFT

Exponential and Logarithmic Functions

Students can describe the relationship between exponential and logarithmic functions.

The student demonstrating acceptable achievement can:

- generate the graph of exponential and logarithmic functions with the use of graphing calculators or graphing utility packages;
- recognize and sketch the graphs of exponential and logarithmic functions and recognize their inverse relationship;
- convert functions from exponential form to logarithmic form and vice versa;
- apply the laws and properties of logarithms to evaluate logarithmic expressions;
- solve and verify simple exponential and logarithmic equations;
- state the domain and range of the exponential and logarithmic functions;
- use the graphs of the exponential and logarithmic functions to estimate the value of one of the variables, given the other variable;
- participate in and contribute toward the problem-solving process for problems that can be represented by logarithmic or exponential functions studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- solve and verify exponential and logarithmic equations;
- complete the solution to problems that can be represented by logarithmic or exponential functions studied in Mathematics 30.

The student demonstrating acceptable achievement can do the following types of questions:

1. An equivalent form of $\frac{3}{4}\log_7(x) = 5$ is

   A. $x^4 = 7^{15}$
   B. $x^3 = 5^{28}$
   C. $x^3 = 20^7$
   * **D.** $x^3 = 7^{20}$

2. The range of $f(x) = 2^x$ is

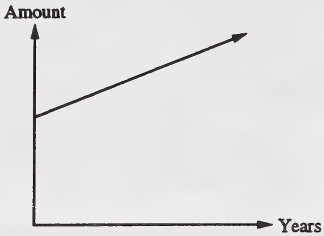
   A. $x \geq 0$
   B. $x > 0$
   C. $f(x) \geq 0$
   * **D.** $f(x) > 0$

3. If $4^{2x} = 90$, then the value of x correct to the nearest tenth is ___[1.6]*.

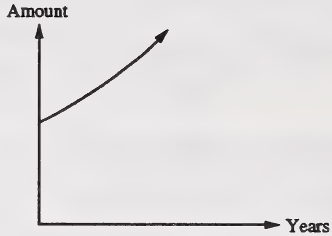
4. If $2^x = 8$, then the value of x correct to the nearest tenth is ___[3.0]*.

5. A radioactive substance decays exponentially so that after four years, half of its original amount remains. The graph that best represents this relation is

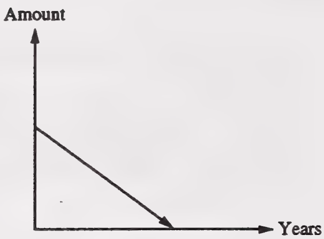
A.



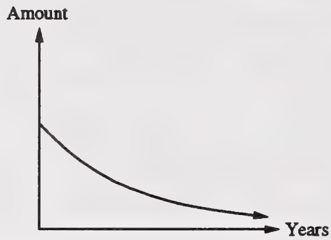
B.



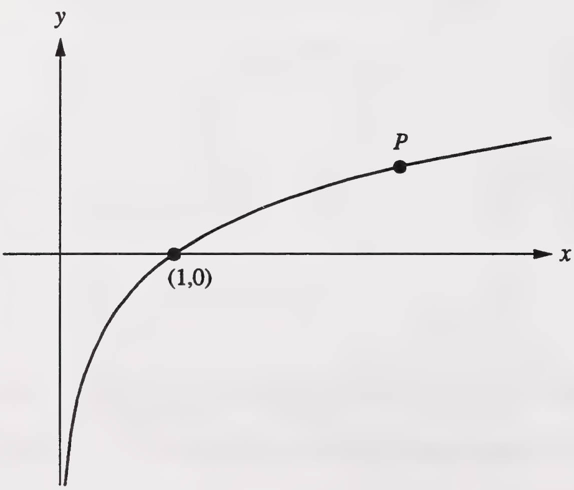
C.



*D.



6. The sketch of the graph of $y = \log_2(x)$ is shown below. If $P(x, 1.54)$ is a point on this graph, then the value of x correct to the nearest hundredth is ___[2.91]*.



The student demonstrating excellent achievement can also do questions such as:

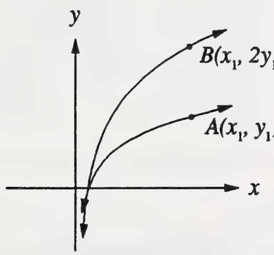
1. If $2\log_{10}(x) + \log_{10}(y) = 3$ and $3\log_{10}(x) - \log_{10}(y) = 7$, then x and y respectively are

   A. 10 and 10
   B. 10 and 0.1
 * **C.** 100 and 0.1
   D. 100 and 10

2. If $3^{2x-5} = 5^{x+1}$, then the value of x correct to the nearest tenth is __[12.1]*.

3. If $\log_{(x-4)}(x^2 - 2x - 61) = 2$, then the value of x correct to the nearest tenth is __[12.8]*.

4. In the diagram below, A is on the graph of $y = \log_7(x)$.



   B is on the graph of

   A. $y = \log_7(2x)$
   B. $y = \log_7(x^{1/2})$
 * **C.** $y = \log_7(x^2)$
   D. $y = (\log_7 x)^2$

The student demonstrating acceptable achievement can *complete* questions such as this written-response question:

5. For the equation $\log_5(x-4) + \log_5(x-2) = \log_5(3)$, find the value of x.

The student demonstrating excellent achievement can also *complete* a question such as:

6. For the equation $\log_5(x-4) + \log_5(x-2) = 3$, find the value of x.

Permutations and Combinations

*All students can describe the difference between a permutation and a combination and calculate the number of permutations or combinations of **n** things taken **r** at a time AND APPLY THESE TO THE EXPANSION OF BINOMIALS.*

The student demonstrating acceptable achievement can:

- calculate the number of linear, circle, and ring permutations and permutations with repetitions of n things taken r at a time;
- calculate the number of combinations of n things taken r at a time;
- expand binomials of the form $(x + a)^n$, $n \in W$ using the Binomial Theorem;
- describe, orally and in writing, the difference between a permutation and a combination;
- participate in and contribute toward the problem-solving process for problems involving permutations and/or combinations, including probability problems studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- expand binomials of the form $(x + By)^n$, $n \in W$ using the binomial theorem and determine specific terms of this expansion;
- explain the reason why there are different numbers of permutations when a given number of objects are arranged in a line, a circle, or in a ring, or when some of the objects are repeated or identical;
- complete the solution to problems involving permutations and/or combinations, including probability problems studied in Mathematics 30.

The student demonstrating acceptable achievement can do the following types of questions:

1. In how many ways can six different mathematics books be arranged on a shelf? [6!]*
2. In how many ways can a committee of four members be selected from a 10-member student council? [${}_{10}C_4$]*
3. In how many ways can seven girls stand in a row if Marissa has to be in the centre? [6!]*
4. In how many ways may Francis make his choice if he is allowed to choose seven out of nine questions on an examination? [${}_{9}C_7$]*
5. Find the coefficient of the x^3 term in the expansion of $(x + 2)^5$. [40]*
6. In the expansion of $(x + y)^7$, how is the coefficient determined in the term containing x^6y? What is the value of the coefficient? [7]*
7. In the expansion of $(x + y)^7$, how many terms are there? How does this relate to the exponent of the binomial? [8, n +1]*
8. What is the probability of getting two heads and one tail if three coins are tossed once? [$\frac{3}{8}$]*
9. In how many different orders can five people sit at a round table? [4!]*

10. Concert organizers are determining the order in which the school bands from Fort McMurray, Grande Prairie, Lethbridge, Red Deer, and Medicine Hat will perform. The number of ways to arrange the order in which the bands will perform if Red Deer performs first is

A. 4
B. 20
* C. 24
D. 120

11. Martha wants to use the digits 2, 3, 4, or 5 for her personal banking identification number. If repetitions are allowed, how many different four-digit numbers can she create?

* A. 256
B. 120
C. 24
D. 16

12. Find the number of possible arrangements of all the letters in the word *curriculum*.
$\left[\frac{10!}{2!2!3!}\right]$*

The student demonstrating excellent achievement can also do questions such as:

1. A Mathematics 30 class was asked to find how many selections of five fruits can be made from five peaches, four pears, two apples, and one grapefruit. What, if any, assumptions must students make when doing this problem? Explain. Do students have enough information to solve this problem?

2. In how many different orders can five people sit at a round table if Jack and Jill must sit next to one another? [12]*

3. Find the coefficient of the x^3y^4 term in the expansion of $(x - 2y)^7$. [560]*

4. How many different arrangements of five letters are possible if two letters are chosen from the word *down* and three letters are chosen from the word *blue*? [${}_4C_3 \bullet {}_4C_2 \bullet 5!$]*

5. Three men and three women are planning to sit at a round table. The group decides on a seating plan that alternates man-woman-man-woman-man-woman. How many such arrangements are possible?

A. 6
* B. 12
C. 18
D. 36

The student demonstrating acceptable achievement can *complete parts a and b* in this written-response question, whereas the student demonstrating excellent achievement can *complete all three parts.*

6. For the high school basketball game, four cheerleaders are working on a special routine.

   a. In how many ways can the four cheerleaders arrange themselves in a row?

   b. In how many ways can the four cheerleaders arrange themselves in a circle?

   c. Explain why there are more ways for the four cheerleaders to arrange themselves in a row than in a circle.

Sequences and Series

Students can describe the differences between sequences and series with an emphasis on arithmetic and geometric sequences, terms of arithmetic and geometric sequences, and can determine the sums of arithmetic and geometric series.

The student demonstrating acceptable achievement can:

- write the specific terms of a sequence given its defining function;
- expand a series given in sigma notation;
- describe, orally and in writing, the difference between sequences and series, arithmetic or geometric, infinite and finite;
- apply the general term formula for arithmetic and geometric sequences;
- apply the sum formula for arithmetic and geometric series;
- participate in and contribute toward the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- solve problems using the general term and/or sum formulas in which there are two unknowns;
- write the specific terms of a sequence given its recursive definition;
- determine the functions that describe any sequence that has a recognizable pattern;
- complete the solution to problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

The student demonstrating acceptable achievement can do the following types of questions:

1. If the sum of the first 16 terms of an arithmetic series is 40 and the common difference is 5, then the first term of this series is

   A. –9
   * **B.** –35
   C. –38
   D. –70

2. The value of $\sum_{n=3}^{6}(-2)^n$ is

* **A.** 40
B. 42
C. 120
D. 126

3. In a geometric sequence, $a = 125$ and $t_4 = 13\ 824$. Correct to the nearest tenth, the common ratio for this sequence is ____[4.8]*.

4. During each 25-year period, an isotope of strontium has its initial mass reduced by a factor of $\frac{1}{2}$. The initial mass of a sample of strontium is 36 mg. The mass of this sample after 325 years is

A. 0.0022 mg
* **B.** 0.0044 mg
C. 0.0088 mg
D. 0.0176 mg

The student demonstrating excellent achievement can also do questions such as:

1. In an arithmetic sequence, $t_4 + t_{13} = 99$ and $t_7 = 39$. The first term of this sequence is

A. –7
* **B.** –3
C. 3
D. 7

2. The nth term of a series is given by $t_n = 5n - 3$. An expression for the sum of n terms of this series is

A. $S_n = \frac{5}{2}(n^2 - n)$

B. $S_n = \frac{5}{2}n^2 - n$

C. $S_n = \frac{5n^2 + n}{2}$

* **D.** $S_n = \frac{5n^2 - n}{2}$

3. A culture of bacteria is being studied in a genetics experiment. The researcher observes that the bacteria double in number every 15 min. After 8 h, the number of bacteria in the culture is N. At this rate, how long will it take for the total population of bacteria to reach $16N$?

* **A.** 9 h

B. $9\frac{1}{4}$ h

C. 16 h

D. 128 h

4. Given the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$ determine the defining function for the nth term.

$[f(n) = \frac{n}{n+1}]$*

The student demonstrating acceptable achievement can *complete* both these written-response questions:

5. An auditorium has eight seats in the first row. Each subsequent row has four more seats than the preceding row.

   a. How many seats are there in the 16th row?

   b. All together, there are 1400 seats in the auditorium. How many rows of seats are there?

6. a. Find the nth term, t_n, of a sequence where the first term, t_1, is 6; the second term, t_2, is 12; and the third term, t_3, is 24.

   b. A sequence different from the one in part (a) has a first term, t_1, of 6 and a third term, t_3, of 24. Find the nth term, t_n.

The student demonstrating excellent achievement can also *complete* this written-response question:

7. Can the number 525 be written as the sum of consecutive numbers? If so, find the sequence. Is there more than one sequence? If so, find the other sequences.

Appendix D

Explanation of Mathematical Abilities

Procedures

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate;
- give reasons for the steps in a procedure;
- reliably and efficiently execute procedures;
- verify the results of procedures empirically (e.g., using models) or analytically;
- recognize correct and incorrect procedures;
- generate new procedures and extend or modify familiar ones;
- appreciate the nature and role of procedures in mathematics.

It is important that students know how to execute mathematical procedures reliably and efficiently; a knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they have given a correct answer; they also must understand concepts underlying a procedure and the logic that justifies it. Procedural knowledge also involves the ability to differentiate those procedures that work from those that do not and the ability to modify them or create new ones. Students must be encouraged to appreciate the nature and role of procedures in mathematics, that is, they should appreciate that procedures are created or generated as tools to meet specific needs in an efficient manner and thus can be extended or modified to fit new situations. The assessment of students' procedural knowledge, therefore, should not be limited to an evaluation of their facility in performing procedures; it should emphasize all the aspects of procedural knowledge addressed in this standard.

Concepts

The assessment of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts;
- identify and generate examples and nonexamples;
- use models, diagrams, and symbols to represent concepts;
- translate from one mode of representation to another;
- recognize the various meanings and interpretations of concepts;
- identify properties of a given concept and recognize conditions that determine a particular concept;
- compare and contrast concepts.

In addition, assessment should provide evidence of the extent to which students have integrated their knowledge of various concepts.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of common examples; it encompasses the broad range of abilities identified in this standard. Assessment, too, must address these aspects of conceptual understanding. Assessment tasks should focus on students' abilities to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and nonexamples, to represent concepts in various ways, and to recognize students' various meanings. Tasks that ask students to apply information about a given concept in novel situations provide strong evidence of students' knowledge and understanding of that concept. Problems designed to elicit information about students' misconceptions can provide information useful in planning or modifying instruction.

Problem Solving

The assessment of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems;
- apply a variety of strategies to solve problems;
- solve problems;
- verify and interpret results;
- generalize solutions.

Students' ability to solve problems develops over time as a result of extended instruction, opportunities to solve many kinds of problems, and encounters with real-world situations. Students' progress should be assessed systematically, deliberately, and continually to effectively influence their confidence and ability to solve problems in various contexts. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments also should yield evidence of students' use of strategies and problem-solving techniques and of their ability to verify and interpret results. Finally, because the power of mathematics is derived, in part, from its generalizability, this aspect of problem solving should be assessed as well.

From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 209, p. 223, p. 228.

Appendix E

Guidelines for Significant Digits, Manipulation of Data, and Rounding in the Mathematics, Chemistry, and Physics Diploma Examinations

Significant Digits

1. For all nonlogarithmic values, regardless of decimal position, any of the digits 1 to 9 is a significant digit; 0 may be significant. For example:

> 123   0.123   0.00230 2.30×10^3
> all have 3 significant digits

2. Leading zeros are not significant. For example:

> 0.12 and 0.012 have two significant digits

3. Trailing zeros to the right of the decimal are significant. For example:

> 0.123 00 and 20.000 have five significant digits

4. Zeros to the right of a whole number are considered to be ambiguous. **The Student Evaluation Branch considers all trailing zeros to be significant.** For example:

> 200 has three significant digits

5. For logarithmic values, such as pH, any digit to the left of the decimal is **not** significant. For example:

> a pH of 1.23 has two significant digits, but a pH of 7 has no significant digits

Manipulation of Data

1. When adding or subtracting measured quantities, the calculated answer should be rounded to the same degree of precision as that of the least precise number used in the computation **if this is the only operation.** For example:

$$\begin{array}{rl} 12.3 & \quad\text{(least precise)} \\ 0.12 & \\ \underline{12.34} & \\ 24.76 & \end{array}$$

The answer should be rounded to 24.8.

2. When multiplying or dividing measured quantities, the calculated answer should be rounded to the same number of significant digits as are contained in the quantity with the fewest number of significant digits **if this is the only operation.** For example:

> (1.23)(54.321) – 66.81483

The answer should be rounded to 66.8.

3. When a series of calculations is performed, the answer should not be rounded off based upon interim values. For example:

> (1.23)(4.321)/(3.45 – 3.21) = 22.145125

The answer should be rounded to 22.1.

Rounding

1. When the first digit to be dropped is less than or equal to 4, the last digit retained should not be changed. For example:

> 1.2345 rounded to three digits is 1.23

2. When the first digit to be dropped is greater than or equal to 5, the last digit retained should be increased by one. For example:

> 12.25 rounded to three digits is 12.3

Appendix F

Directing Words

Discuss
The word "discuss" will not be used as a directing word on math and science diploma examinations because it is not used consistently to mean a single activity. It can mean to debate, i.e., present arguments both pro and con; to investigate, i.e., present in detail the factual information about a topic; or simply to talk about a subject.

The following words are more specific in meaning.

Contrast/Distinguish
Point out the *differences* between two things that have similar or comparable natures.

Compare
Show the character or relative values of two things by pointing out their *similarities* and *differences.*

Conclude
State a logical end based on reasoning and/or evidence.

Criticize
Point out the *merits* and *demerits* of an item or issue.

Define
Provide the essential qualities or meaning of a word or concept. To make distinct and clear by marking out the limits.

Describe
Give an account of in words; or represent by a figure, model, or picture the characteristics.

Design/Plan
Construct a plan, i.e, detailed sequence of actions, for a specific purpose.

Enumerate
Specify one by one or list in concise form and according to some order.

Evaluate
Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.

Explain
Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.

How
Show in what manner.

Hypothesize
Form a tentative proposition intended as a possible explanation for an observed phenomenon, i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.

Identify
Recognize and select as having characteristics of.

Illustrate
Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram.

Infer
Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.

Interpret
Tell the meaning of; present information in a new form that adds meaning to the original data.

Justify/Show How
Show reasons for, or give facts that support a position.

Outline
Give, in an organized fashion, the essential parts of. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps.

Predict
Tell in advance on the basis of empirical evidence and/or logic.

Prove
Establish the truth, validity, or genuineness of something by giving factual evidence or logical reasons.

Relate
Show logical or causal connection between.

Solve
Give a solution, i.e., explanation in words and/or numbers, for a problem.

Summarize
Give a brief account of the main points.

Trace
Give a step-by-step description of the development of.

Why
Show the cause, reason, or purpose for which.

Appendix G

Preparing to Write a Mathematics 30 Diploma Examination – What every student should plan for before the examination!

1. Prepare a course review schedule:
   - design your schedule for a two-week period (minimum) before the examination schedule
   - divide the course material into sections and indicate on the schedule the time blocks to be devoted to each section
   - take into account the examination blueprint available from your teacher (*Mathematics 30 Information Bulletin, Diploma Examinations Program)*. Note that course units are not equally weighted on the diploma examination
   - take into account units/concepts that you find most difficult; i.e., allocate more time for the review of these

2. Obtain and review examination schedules, rules, and policies:
   - record time and place of writing
   - note minimum and maximum writing times permitted
   - prepare for remaining in the examination room for at least 2.5 h (Kleenex, cough drops, etc.),
   - identify materials allowed for writing each examination: pencils, pens, calculators, and mathematical instruments

3. Identify and collect examples of each type of question that will be asked:
   - obtain a copy of the relevant information contained in the *Mathematics 30 Information Bulletin, Diploma Examinations Program* (available from your teacher)
   - review the format of the previous diploma examination (available from your teacher)
   - learn the meaning of key "directing" words; e.g., describe, explain, interpret, illustrate, compare, evaluate, prove/justify

4. Make summaries and point form outlines:
   - distinguish between major concepts and factual details
   - identify essential skills that can be assessed on paper and pencil tests
   - review lab results and procedures – identify connections between lab reports, class notes, and textbook
   - anticipate examples of connections between concepts and the "real world"
   - prepare a glossary of important subject terminology
   - review the formula sheet, z-score page, and 90% Box Plots for Mathematics 30
   - link each formula or equation with a calculation
   - identify any restriction on the use of each formula or equation

5. Use memory aids such as:
   - color coding, underlining, highlighting, jotting key words in margins
   - grouping word and idea associations
   - reading aloud key words, expressing key words in your own words

Appendix H

Suggestions for Students Writing Mathematics 30 Diploma Examinations – What every student should know when writing examinations!

1. Do not be afraid to answer each question even if you are not sure of the correct solution to the problem. A penalty is NOT given for guessing on the machine-scored section the exam. Partial marks are often awarded for incomplete answers in the written-response section of the exam.

2. If you are stuck on a question, mark the alternatives that you know are incorrect and choose from the ones that are left using a logical guessing strategy. Think of the questions as challenges and cultivate a positive attitude about your ability to answer them.

3. Scan the written-response and multiple-choice sections of the examination before answering the questions. A question in one section of the examination may jog your memory about a question in another section.

4. When first reading a multiple-choice question, locate and circle key words to help clarify the meaning of the question. Then hide the alternatives and try to formulate an answer of your own. Your answer may be very close to the correct alternative.

5. If a multiple-choice question involves a calculation, do the calculation and select the alternative that is closest to your answer. A multiple-choice calculation is usually short. If you cannot get it in five minutes, your method is either inappropriate or incorrect. Go on.

6. Diagrams on examinations are often labeled with numbers or letters. It may be useful to write in the names of the labeled structures or features that you can identify.

7. When reading graphs, use a clear plastic ruler to extrapolate or interpolate data more accurately.

8. Have a good reason for changing an answer. Do not change an answer on a hunch. Do not waste your time looking for patterns of A, B, C, or D in multiple-choice answers. There are none.

9. You may not have time to write and edit a complete rough copy for each written-response question, but you should prepare an outline of your answer and use it as a guide when writing your good copy.

10. When completing a written-response question, keep in mind the reader of your response. The reader will want to know how well you:
    - understand the problem or the mathematical concept
    - can correctly use the mathematics involved
    - can use problem-solving strategies and explain your answer and procedures
    - can communicate your solutions and mathematical ideas

11. Rewriting a statement of the question is often a good way to begin a written response. Conclude with a summary statement. Be sure you have clearly explained all assumptions and have verified your conclusions.

12. Keep track of the time and pace yourself. Put a check mark by items that you are uncertain about and return to them if there is time at the end of the examination.